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SWAP PRICING

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Abstract

A financial Swap transaction typically involves an agreement to exchange financial assets today and re-exchange those in the termination o the swap. In most swaps, the parties repay any income on the asset they received.

The swap market developed because two different investors would find that while one of them had a comparative advantage in borrowing in one market, he was at a disadvantage in the particular market in which he wanted to borrow. If these markets were counter-matched by the two parties with their relative advantages, the two could get the best of both worlds through a swap.

Financial derivatives afford numerous benefits to both speculators and risk managers. In addition, derivatives offer some surprising advantages in reducing transaction costs and other forms of trading efficiency.

Throughout this project, we will be concerned with the principles that determine the prices of the financial derivatives, which we consider, are critically important in finance today and play a key role in virtually all-financial markets.

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1. Introduction

The swap market is a liquid market, but few people have the right tools to accurately price their existing positions.

Modern risk management does not make the traditional distinction between pure and statistic risk. In addition to the pure risks all organizations encounter, many business firms, especially financial firms as banks, face a whole set of contingencies we call financial risk.

Many firms consider managing financial risk to be a separate problem form managing other loss exposures; these firms do not manage their financial risks in the same departments as their pure risks. Other firms take a broader view. They see that cost of capital issues and cash flow management issues are common threads connecting financial and pure risk management

During the last decade credit, derivatives market has grow exponentially, and it has become important instruments to lay off or trade on credit risk.

Companies doing business in more than one country need to protect them themselves against sudden or dramatic changes in the relative value of currencies, so they hedge commitments to invest, sell or borrow with agreements that have predetermined forex, or foreign-exchange, rates.

A swap contract involves the risk that one or either party to the swap may default. This default or credit risk is often referred to as counter party risk that is a major concern to financial institutions and regulatory bodies.

2. Swaps

Swap are agreements between two parties where the first pays a floating rate to the second, while the second pays a fixed rate to the first, with both payments being based on a common principal amount.

The agreement defines the dates when the cash flows are to be paid and the way that they are to be calculated. Usually the calculation of the cash flows involves the future values of one or more market variables.

Forward transaction mean agreeing on an exchange rate that will apply when currency is trade on asset date in the future.

Forward contract leads so the exchange of cash flows on just one future date, swaps typically lead to cash flow exchanges on several future dates.

The payments are made each period during an interval of time. The floating rate payment is based on the spot rate *r*, with the value actually used being either the one for the period just ended or the one for the period about begin, (swap settled in arrears or settled in advance respectably.

The first swap contracts were negotiated in the early 1980s. Since then the market has grow very rapidly. Hundreds of billions of dollars of contracts are currently negotiated each year.

2.1 Swap Terminology

Principal Amount	The principal Face value of the swap. This may be notional amount, as in an interest rate swap, or it actually is exchange in a currency swap.
Fixed rate	The fixed interest rate paid over the life of a swap
Floating rate	The variable interest rate on a swap, usually expressed in the term of a money market reference interest rate such as LIBOR.
Fixed rate payer	The party that pays a fixed rate and receives the floating rate in an interest rate swap.
Fixed rate receiver	The party that receives the fixed rate and pays the floating rate in an interest rate swap.
Trade date	The date on which a swap contract is executed
Valuation date	The date on which a valuation being performed.
Effective date or start date	The date on which the swap contract commences and this date that the initial exchanges will take place, the interest accrual will start.
Payment frequency	The frequency with which interest payments are calculated.
Payment date	A list of all the dates on which interest will paid against the swap
Termination or maturity date	The date on which the swap terminates. All interest accrual s stop and any exchange of the principal will take place on this date.

Table 1 Swap terminology and Contract.

3. Interest Rate Swaps

The common type of swap is a "plain vanilla" interest rate swap.
In this, company B agrees to pay company A cash flows equal to interest at a predetermined fixed rate on a notional principal for a number of years. At the same time, company A agrees to pay company B cash flows equal to interest at a floating rate on the same notional principal for the same period of time. The currencies of the two sets of cash flows are the same.

3.1 Hypothetical Plain Vanilla Swap

The subsequent table specifies that the following business day convention is to be used and that the U.S. calendar determines which days are business days and which days are holidays.
This means that, if a payment date falls on a weekend or a U.S. holiday, the payment is made on the next business day.

Trade Date Effective Date Business day convention (all days) Holiday calendar Termination date	27-Feb-2003 5-March-2003 Following business day U.S. 5-March-2006
Fixed Amount Fixed-rate prayer Fixed rate notional Principal Fixed rate Fixed-rate day count conventions Fixed rate payments dates	Microsoft U.S. $ 100 Million 5% per annum Actual / 365 Each 5-march and 5- September commencing 5-September-2003 up to and including 5-March -2006
Floating Amount Floating-rate prayer Floating rate notional Principal Floating rate Floating-rate day count conventions Floating rate payments dates	Microsoft U.S. $ 100 Million U.S. $ 6-Months LIBOR Actual / 360 Each 5-March and 5- September commencing 5-September-2003 up to and including 5-March -2006

Table 2 in this example September 5, 2004 is a Sunday. The payment is therefore made on Monday September 6, 2004.

3.2 London Interbank Offer Rate (LIBOR)

The floating rate in many interest rate swap agreements is the London Interbank Offer rate that is, the rate at which large international banks fund much of their activities.

LIBOR is the rate at which one large international bank is willing to lend money to another large international bank. The rate is determined in trading between banks an change as economic conditions change.

LIBOR is a reference rate of interest loans in international financial markets.

3.3 Pricing Schedule

The price of SWAP will depend on the assets being swapped. Determining the present value of swap is comprised of two steps.

- Pricing the SWAP: Obtaining the current prices and interest rates that the market is willing to enter into a SWAP transaction.

- Valuing a SWAP: Applying the market prices to an outstanding swap to determine the amount of cash or present value that would have to be paid or receive to terminate the swap.

Looking at the components of Swap is also useful in highlighting where the value to swap user can arise:

- Arbitrage: If the value of a SWAP is not the sum or equivalent of its cash market components, then an arbitrage opportunity exists. The evolution of SWAP market can b attributed to this type of mis-pricing between swap and cash market.

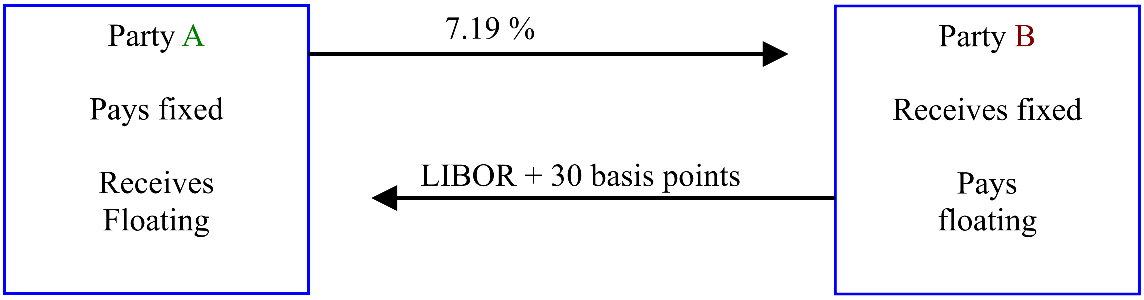
- Low cost: A swap transaction creates an asset and liability at a considerably lower transaction cost and in much shorter time than creating the equivalent physical asset and liability

- Off Balance Sheet: A swap transaction not considered as balance sheet item and as such does not build the balance sheet in the same way as the equivalent asset and liability.

- Default risk: In terms of pricing, the nature of this default risk needs to be incorporated into our model. While the default of any instrument, like bond, arised huge loss, the swap will incorporate only a minimal component reflecting the risk of default.

Consider the following swap. Party A pays a fixed rate 7.19 percent per annum on a semi-annual basis, and receives from party B LIBOR + 30 basis points.

We assume that the notional Principal $ 35,000,000. Days in Period 182.The fixed rate in a swap is usually quoted on a semi-annual bond equivalent yield basis. The current six-months LIBOR rate is 6.45 percent per annum.



The amount that's paid every six months is \$ 1,254,802.74

$$\left(35{,}000{,}000\right)\left(\frac{182}{365}\right)\left(\frac{7.19}{100}\right)=\$1{,}254{,}802{,}74$$

The floating side is quoted on a money market yield basis. The difference between the two rate computations is in the number of days in a year convention employed[1].

$$\left(35{,}000{,}000\right)\left(\frac{182}{365}\right)\left(\frac{6.45+0.30}{100}\right)=\$1{,}194{,}375$$

In a swap, the payments are netted. In this case, Party A pays Party B the net difference

$$\$\ 1{,}254{,}802{,}74-\$1{,}194{,}375=\$\ 60{,}427.74$$

Conclusions:

1st Party A sent Party B a payment for the net amount.

2nd Principal isn't exchange.

3rd Party A is exposed to the risk that Party B might default, and conversely, Party B is exposed to the risk of Party A defaulting[2].

4th On the fixed-payment side a 365-day year is assumed, while on the floating payment side a 360-day is used[3].

3.3.1 Pricing of Plain Vanilla Swap

A plain vanilla interest rate swap with a notional principal for a tenor of two years and payments every six months is denoted by

NP= Notional Principal.
FR $A_{x,y}$, is the interest rate on an FRA's (Forward Rates Agreements) for the period beginning at $t=x$ and extended until time $t=y$.

[1] The money market yield can be converted to a semi-annual bond equivalent yield.
[2] If one party defaults, the swap usually terminates.
[3] The number of days in the year is one of the issues specified in the swap contract.

PART = fraction of the year between swap payments.

Zx, y = Zero coupon factor for an investment initiated at time x and extending until time y.

SFR = Unknown Swap Fix Rate.

According to no-arbitrage principal, the sequences of fixed and floating payments must have the same present value at the initiation of the SWAP. Hence, for no-arbitrage condition we can write

$$\frac{FRA_{0,6} \times PART \times NP}{Z_{0,6}} + \frac{FRA_{6,12} \times PART \times NP}{Z_{6,12}} + \frac{FRA_{12,18} \times PART \times NP}{Z_{12,18}} + \frac{FRA_{18,24} \times PART \times NP}{Z_{18,24}} =$$

$$= \frac{SFR \times PART \times NP}{Z_{0,6}} + \frac{SFR \times PART \times NP}{Z_{6,12}} + \frac{SFR \times PART \times NP}{Z_{12,18}} + \frac{SFR \times PART \times NP}{Z_{18,24}}$$

$$SFR = \frac{\frac{FRA_{0,6}}{Z_{0,6}} + \frac{FRA_{6,12}}{Z_{6,12}} + \frac{FRA_{12,18}}{Z_{12,18}} + \frac{FRA_{18,24}}{Z_{18,24}}}{\frac{1}{Z_{0,6}} + \frac{1}{Z_{6,12}} + \frac{1}{Z_{12,18}} + \frac{1}{Z_{18,24}}}$$

The above equation provides a generalised form for the swap fixed rate on a plain vanilla interest rate swap, if it has total of N payments, occurring at a regular interval of months apart:

$$SFR = \frac{\sum_{n=1}^{N} \frac{FRA_{(n-1)M,\, nM}}{Z_{(n-1)M,\, nM}}}{\sum_{n=1}^{N} \frac{1}{Z_{(n-1)M,\, nM}}}$$

3.4 Valuation of Interest Rate Swaps

Interest Rate Swap (IRS) is one of the most important and widely traded instruments in the global derivatives market, and is a normal derivation of the fixed income market.

IRS is an exchange of cash flows depending on different conditions, schedule payments and dates. The most popular transaction of IRS is the exchange of fixed rate flows for floating rate flows or vice-versa. The basic assumption of IRS is that there is no exchange of the principal notional amount among parties. Only interest related cash flows are exchanged.

Swap valuation involves calculating the present value of all remaining futures cash flows created by swap. So after a swap has been executed and the initial exchange has already taken place, or was treated as notional exchange, then valuation will take into account all the remaining interest and principal cash flows.

There are number methods of generating swap present value. However, they can be divided into two general approaches:

- The bond method: Under this method we break a swap into its underlying components described in general terms as bonds and then calculate the present value of each cash flow in a similar manner to the bond valuation formula.

- The offset method: In this case, an existing swap is valued by offsetting it with a swap starting today. This creates a stream of futures cash flows that are then present valued. The offset method actually determines the difference between the existing swap and a swap starting today and then converts these to a present value amount.

In a zero-coupon interest rate swap, payments only occur at maturity, at which time one counterparty pays the total compounded fixed rate over the life of the swap and the other pays the total compounded floating rate that would have been earned had a series of LIBOR investments been rolled over through the life of the swap.

The swap spread is determined by market trends, and is differentiated according to the creditworthiness of the borrowers. Generally, the swap is priced at-the-money to start with, which means the price where the value of floating rate cash flows is equal to the value of fixed rate cash flows. The relative value of the IRS varies as the interest rate fluctuates.

There are several steps to calculate the value of an IRS in the market:

1) Recognise and classify the cash flows.

2) Form a swap curve, with the help of the Government yield curve and swap spread curve.

3) Create a zero-coupon curve with the help of swap curve.

4) Calculate the present of value of cash flows using zero-coupon rates.

Interest rate Swaps create a stream of future interest cash flows. One leg, the fixed rate has known interest cash flows, until the swap maturity. While the interest on the other, floating leg is only known until the next interest payment date. As with any derivatives contract, valuation involves converting both the known and unknown future cash flows to single, known, cash value today. There are many models to get the valuation of swap. In this project, we will present two models.

3.4.1 Simple Offset Model

This valuation method generates the present value of a swap by offsetting an outstanding swap with an offsetting swap executed at prevailing market interest rate. The cash flows of the original and offset swap take place on the same day so the floating-rate payments will offset and fixed-rate payment will leave a residual according to the difference in the fixed interest rates.

The value of the swap is calculated by present valuing these residual cash flows.

The Formula can be summarised as follows: the value of swap from the fixed receiver's point of view on an interest payment date:

$$PV_{rec} = [(r-r^*)/(1+r^*/m)+\ldots.+(r-r^*)/(1+r^*/m)^n + (r_1^* - r_1)/(1+r^*/m)]\times FV$$
$$PV_{pay} = Value_{rec} \times -1$$

This formula assumes even interest periods and compound interest periods and compound interest rates where:

r = original swap yield to maturity in original swap (%pa).
r^* = current swap yield to maturity in offsetting swap (%pa).
r_1 = original floating yield to next interest payment date (%pa).
r_1^* = current floating yield to next interest payment date (%pa).
n = the number of periods to maturity excluding the next n payment date.
m = the number of interest payments pa.
FV = the notional face value of the swap.

3.4.2 Single Rate bond method

In this case, we make use of the synthetic replication of a swap and break the two legs into two bonds –one with a fixed interest rate and one with a floating interest rate.

The current value of these bonds is calculated and the value of the swap is determined by the net value of these two instruments. It is called the single rate method because the fixed leg is present valued with just yield –the prevailing swap rate for the remaining term to maturity.

The value of swap is net of the underlying fixed and floating bonds on the an interest payment date:

$$PV_{rec} = PV_{fixed} - PV_{floating}$$
$$PV_{pay} = Value_{rec} \times -1$$

Where:

$$PV_{fixed} = [r/m \times (1+a_n)+v^n)]\times FV$$
$$PV_{floating} = [(r_1/m+1)/(1+r_1^*/m)]\times FV$$

where:

r= original swap yield to maturity in original swap (%pa).
r^*= current swap yield to maturity in offsetting swap (%pa).
r_1= original floating yield to next interest payment date (%pa).
r_1^*=current floating yield to next interest payment date (%pa).
n=the number of periods to maturity excluding the next n payment date.
m= The number of coupon payments pa.

$v=1/(1+r^*/m)$.
$a_n=(1-v^n)/(r^*/m)$.
FV=the notional face value of the swap.

3.5 Swaps and Forward

Swaps have similar features to forwards. After the initial exchange has taken the place, the commitment to re-exchange principal amount s at maturity is similar to a forward agreement. Instead of cost carry being incorporated into the for ward price, the net cost or benefit of the final exchange is paid during the life of the swap.

As well as being decomposed into cash instrument swaps can also be decomposed into series of forward agreement.

This is easiest to explain with an example: suppose we enter into a 1 year US$ 10 million equity swap under which you receive the dividends of the underlying stock annually and pay 12 month LIBOR rate. At time to maturity, we will receive the underlying stock and pay the 12-month LIBOR rate. There is no initial exchange under the swap.

Suppose the LIBOR rate is 8% pa. And dividend rate is agreed at 4% pa. The cash flows of the swap will be as follows:

DATE	ITEM	Cash flows
Today	Start swap	$ 0
1 year	Receive Dividend	$ 400,000
	Pay LIBOR	-$ 800,000
	Pay for shares	-$ 10,000,000
	Net cash flow	-$ 10,400,000

As we can see, the cash flows of this transaction are the same as 1-year forward purchase of the underlying stock. If we entered into forward agreement, we would agree to purchase the stock as a forward price as follows.

Forward price	=	Spot price x (1+(r - q) x f / D
	=	10,000,000x(1+(0.08-0.04) x 365/365
	=	10,400,000

The swap and the forward have identical outcomes. The only difference is in way of the price quoted. The SWAP price is expressed in the terms of the dividend yield and the interest rate, while the forward price is expressed as the final purchase price.

3.6 An Example of Interest Rate Swaps

Company A pays to Company B the six months LIBOR rate 4.2% per annum.
Company B pays to Company A 5% interest per annum.
The Notional Principal is $ 100,000,000.
Period is 2, % semi-annual compounding.
Three-years tenor.

Date	Libor Rate %	Floating CashFlow Received	Fixed Cash Flow Paid	Net Cash-Flow
03-01-1999	4.2			
09-01-1999	4.8	+ 2.10	- 2.5	- 0.4
03-01-2000	5.3	+ 2.4	- 2.5	- 0.1
09-01-2000	5.5	+ 2.65	- 2.5	+ 0.15
03-01-2001	5.6	+ 2.75	- 2.5	+ 0.25
09-01-2001	5.9	+ 2.8	- 2.5	+ 0.3
03-01-2002	6.4	+ 102.95	- 102.5	+ 0.45

Table3 Cash Flow to Company B (Millions of Dollars)

4. Foreign Currency Swaps

Multinational firms needing to finance their operations should issue different securities to investors in different countries in order to aggregate their disparate information about domestic and foreign cash flows.

Currency swaps involves exchanging principal and interest payment in one currency for principal and interest payment in another currency. The agreement requires that a principal be specified in each of the two currencies.

The principal amounts are usually exchanged at the beginning and at the end of the life of the swap. The principal amounts are chosen to be approximately equivalent using the exchange rate at the time the swap is initiated.

If the firm becomes bankrupt, investors may face uncertain costs of reorganizing assets in a foreign country and thus may value foreign assets at their average value. This penalizes superior firms with low reorganization costs, such firms minimize the adverse selection penalty by designing securities that allocate all the cash flow in bankruptcy to investors for which the adverse selection costs are the smallest given the exchange rate.
This sharing rule can be implemented with currency swaps because these instruments allow the priorities of claims in bankruptcy to switch depending on the exchange rate.

Currency swaps are an offshoot of the IRS, when different currencies are involved. The swap of interest, sometimes involves the exchange of principals at the inception as well as at maturity. The exchange of principal and interest rate is dependent on prevailing market rates.

4.1 Currency Swap Pricing

An effective swap model must be able to incorporate four possible combinations. For example, party C may have German marks and be anxious to swap those marks for US dollars. Similarly, Party D may hold U.S. dollars and be willing to exchange those dollars for German marks. Therefore, the combinations may be as follows.

1. Party C pays a fixed rate on dollars received and Party D pays a fixed rate on marks received.

2. Party C pays a floating rate on dollars received and Party D pays a fixed rate on marks received.

3. Party C pays a fixed rate on dollars received and Party D pays a floating rate on marks received.

4. Party C pays a floating rate on dollars received and Party D pays a floating rate on marks received.

4.2 Currency Swap Example

Principal amounts are $15,000,000 and £10,000,000.
Five-years tenor.
Initiated date February 1, 1999.
Interest payments are made once a year.

Company **A** pay fixed interest rate of 11% in Sterling pound and receives a fixed interest rate of 8% in dollars.

Date	Dollar Cash Flow	Sterling Cash Flow
February 1,1999	- 15	+ 10
February 1,2000	+ 1.20 (8%*$15)	- 1.10 (11%*£10)
February 1,2001	+ 1.20	- 1.10
February 1,2002	+ 1.20	- 1.10
February 1,2003	+ 1.20	- 1.10
February 1,2004	+ 16.20	- 11.10

Table 4 Cash Flows to Company A (Millions of Dollars)

4.2.1 Advantages and Disadvantages of Currency Swap

Currency swaps help to explore global capital markets efficiently, and plays a significant role in the burgeoning market of securitisation and diversification of portfolios. It helps to leverage comparative advantage of borrowing in different countries.

The potential exposure is very high due to exchange and re-exchange in different currencies. Volatility will increase along with increase in time.

4.3 Valuation of Currency Swaps

In the absence of default risk, a currency swap can be discomposed into a position in two bonds, as is the case with and interest rate swap

$$V_{swap} = B_D - S_O\, B_F$$

V_{swap} = is the value in US dollar of a swap.
B_D = is the value in US dollars[4].
S_O = is the current spot exchange rate[5].
BF = is the value measured in foreign currency[6].

The Value of the swap can be determined from LIBOR rates in the two currencies and the spot exchange rate.

The value of the a swap where the foreign currency is received and dollar are paid is

$$V_{swap} = S_O\; B_F - \; B_D$$

Consider a forward contract that expires at date T. The value in terms of U.S.$ of this forward contract at date T is

$$V(T) = S(T) - F(0,T)$$

Where S (T) is the dollar/GBP spot exchange rate at date T and F (0, T) is the dollar/GBP forward exchange.

The Value of the forward contract in USD at date 0 is therefore

$$V(0) = PV_0\,\left[\,S(T) - F(0,T)\,\right]$$

$$V(0) = PV_0\,\left[\,S(T)\;\;\right] - F(0,T)\,B_D\,\left(\,0,T\;\right)\big],$$

By Market convention the forward exchange rate $F(0,T)$ is set such that the initial value of the forward contract is zero. $V(0) = 0$
Which implies, $PV_0\,[S(T)] = F(0,T)\,B_D\,\left(\,0,T\,\right)$.

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4 Of the US dollar bond underlying the swap
5 Expressed as, number of dollar per unit of foreign currency.
6 Of the foreign denominated bond underlying

Now consider the foreign currency swap of USD for GBP, where:

N_D denotes the USD principal ,
N_F denotes the GBP principal.
R_D is the fixed USD rate
R_F is the fixed GBP rate

Suppose payment occur at date $t_{1,} t_{2,\ldots,} t_n$, where $t_{n,}$ is the maturity of the swap.

At date $t_{1,}$, the net payment is $R_D N_D - S(t_1) R_F N_F$, assuming dollar as numeraire.

The Present USD value of this payment $t = 0$ is $B_D(0, t_1) R_D N_D - PV_0[S(t_1)] R_F N_F$

Substituting Expression $PV_0[S(T)] = F(0,T)\, B_D\,(0,T)$ into the expression above, gives

$$B_D(0, t_1) R_D N_D - F(0, t_1)\, B_D(0, t_1)\, R_D N_D$$

Repeating the argument for the remaining payments gives the U.S.$ value of the swap at date 0,

$$V_{SWAP} = \sum_{j=1}^{n} B_D(0,t)\left[R_D N_D - F(0,t_j) R_\$ N_\$\right] + \underbrace{B_F(0,t_n)\left[N_D - F(0,t_n) N_F\right]}$$

The payment of principals at date $t_{n,}$

This is the expression of valuing of the plain Vanilla foreign currency swap to counterparty A in terms of forward exchange rates and Zero-coupon bond prices, all of which are observable at date 0.

5. Commodity Swaps

In a typical commodity swap, one counter party makes periodic payments to the second counterparty at a fixed price per unit for a given notional quantity of some commodity. The second counterparty pays the first counterparty a floating price per unit for a given notional quantity of some commodity.

The commodities are usually the same. The floating price is usually defined as an average price, the average being calculated using spot commodity prices over some predefined period.

Commodity swaps are becoming increasingly common in the energy and agricultural industries, where demand and supply are both subject to considerable uncertainty.

5.1 Types of commodity swaps.

There are two types of commodity swaps namely:

- Fixed floating commodity swap
- Commodity for interest swap

Fixed floating swaps are just as the fixed-floating swaps in the interest rate swap market, with the exception that both indices are commodity based indexes.

Commodity for interest swap is similar to the equity swap in which a total return on the commodity in question is exchanged for some money market rate (plus or minus a spread).

5.2 Valuation of Commodity Swaps

In pricing commodity swaps, we can think of the swap as a strip of forwards each priced at inception with zero market value (in a present value sense).

In valuing commodity swaps, we must be able to account minimum the following: The cost of hedging. The liquidity of the underlying commodity market. The variability of the futures bid. The credit risk

These peculiarities refer more to the often-limited number of participants in these markets, the unique factor driving these markets and the individual participants in these markets

A plain vanilla commodity swap is one in which counterparty A agrees to pay counterparty B a fixed amount $X per unit of the commodity at dates $t_1, t_2, \ldots, t_n$ where t_n is the maturity of the swap. The notional principal is Np units of the commodity. Counter party B agrees to pay counter party A the spot price of the commodity at dates $t_1, t_2, \ldots, t_n$.

We now value this commodity swap on the first payment at time t_1. The net payment to counter party A at date t_1 is $[S(t_1) - X] N_p$.
Where:

$S(t_1)$ is the spot price of the commodity at time t_1.

X is the unit of the commodity.

NP is the notional principal units of the commodity.

The value of this payment at date 0 is $\{ PV_0 [S(T1)] - XB(0, t_1) \} Np$

In the absence of carrying cost and convenience yields, the present value of the spot price $S(t_1)$ would be equal to the current spot price. In practice, there are carrying cost and convenience yields.

5.3 Commodity Swap Example

Heavy users of oil, such as airlines will often enter into contracts in which they agree to make a series of fixed payments, say every six months for two years, and receive payments on those same dates as determined by an oil price index. The calculation of the demand are often based on a specific number of tons of oil in order to lock in the price the airline pays for a specific quantity of oil, purchased at regular intervals over the two year-period.

We note that in most interest rate, currency and equity swaps, the variable payment is based on the price or rate on a specific day, whereas in swaps like that of oil, the variable payment is based on the average value of an oil index over a period of time. This eliminates the effect of an unvolatile simple day and ensures that the payment will be more accurately represented as usual value of the index.

A Company has a constant demand for 50,000 barrels of oil per month and is concerned about volatile oil prices. It enters into a three-years commodity swap with a swap dealer.

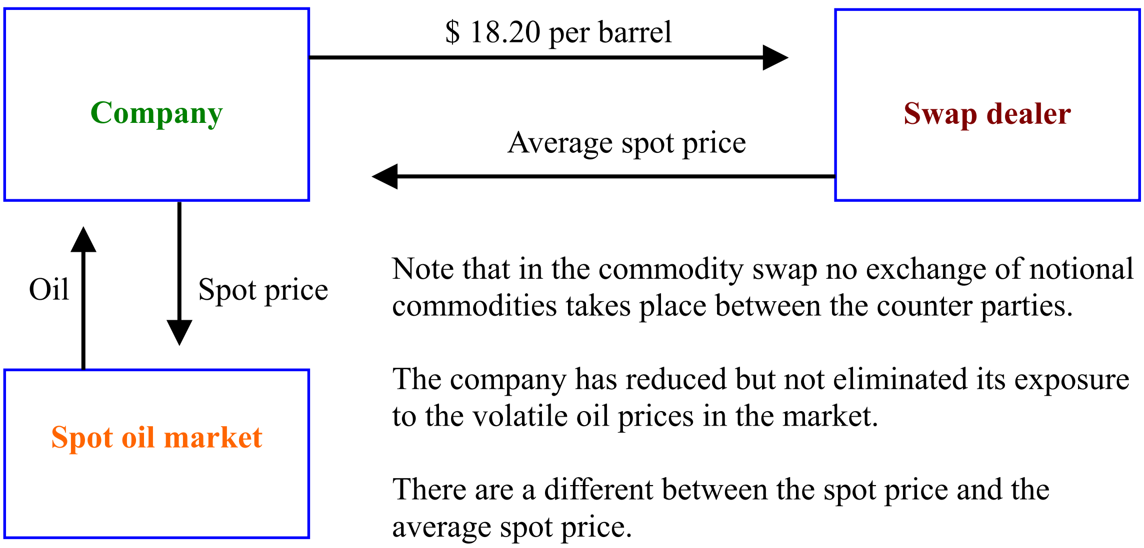
The current spot oil price is \$ 18.10 per barrel. The Company agrees to make monthly payments to the swap dealer at a rate of \$ 18.20 per barrel.

The notional principal is 50,000 barrels.

The swap dealer agrees to pay the Company the average daily price for oil during the preceding month.

The Company has buying oil and paying the spot price and also paying to the swap dealer \$ 18.20 per barrel over the life of the contract, and from the swap dealer it receives the last month's average spot price.

Representation the problem in diagram:



Note that in the commodity swap no exchange of notional commodities takes place between the counter parties.

The company has reduced but not eliminated its exposure to the volatile oil prices in the market.

There are a different between the spot price and the average spot price.

6. Equity Swap

An Equity swap is an agreement to exchange the dividends and capital gains realized on an equity index for either a fixed or a floating rate of interest. The Swaps are arranged base on some notional face value at the start of the swap and there is a regular exchange of cash flows based on an agreement term to maturity

Equity swaps can be used by portfolio managers to convert returns from a fixed or floating investment to the returns from investments in an equity index, or vice versa.

Equity swaps are used in index trading. This passive investing strategy is gaining ground in the fund management community. Instead of trying to buy individual stocks that are deemed to be undervalued by some method of fundamental analysis, the index trading mechanism chooses a basket of stocks that is selected its ability to represent the general market or one particular sector of the stock market.

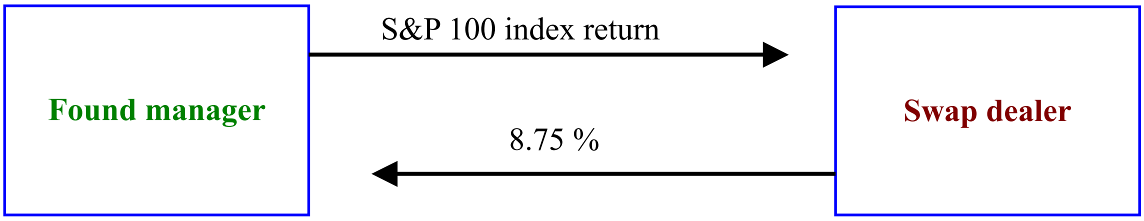
The fees associated with funds that engage in index trading are much lower because the investment management is mechanically deterministic. It is prescribed by the index that the investors have chosen.

6.1 Equity Swap Example

Consider the portfolio of an equity fund the return of which is highly correlated to the S&P 100 stock index. The fund manager is concerned about the risk exposure and decides to enter into an equity swap.

The found manager agrees to pay the swap dealer the S&P 100 return and receive from the swap dealer a fixed rate of 8.75 percent per annum. These payment are to be med quarterly and the notional principal is fixed at 100 million dollar

Illustration the problem in diagram:



The found manager can alter the risk exposure of the found by using and equity swap.

The found manager transformed an equity portfolio into a risk less return of 8.75 %, ignoring counterparty risk.

The return on the index can be positive or negative. If it's positive, the found manager pays to the swap dealer, if it's negative, the swap dealer pays to the found manager.

7. Matlab Application

Term Structure Analysis and Interest Rate Swap Pricing

This example illustrates some of the term-structure analysis functions, the syntax of the program was found in the Financial Toolbox.

The forward curve implied from the market data are then used to price an interest rate swap agreement.

```
% File: Swap.m
% I illustrate forward curves from the observed market prices of
% coupon-bearing bonds, which is used to price an interest rate swap agreement.
% I specified the value for the settlement date, Maturity dates, Coupon Rates,
% and Market prices for 10 U.S. Treasury Bonds.
% I price a five-years tenor.

% Author: Rafael Vides.

Settle=datenum('15-Jan-1999');
BondData={'15-Jul-1999' 0.07000 99.93
'15-Jan-2000' 0.07125 99.72
'15-Jul-2000' 0.07375 99.70
'15-Jan-2001' 0.07500 99.40
'15-Jul-2001' 0.07875 99.73
'15-Jan-2002' 0.08000 99.42
'15-Jul-2002' 0.08250 99.32
'15-Jan-2003' 0.08375 99.45
'15-Jul-2003' 0.08500 99.71
'15-Jan-2004' 0.09000 99.15};

Maturity=datenum(strvcat(BondData{:,1}));
CouponRate=[BondData{:,2}]';
Prices=[BondData{:,3}]';
Period=2;
ZeroRates=zbtprice([Maturity CouponRate], Prices, Settle);
ForwardRates=zero2fwd(ZeroRates, Maturity, Settle);
DiscountFactors=zero2disc(ZeroRates, Maturity,Settle);
PresentValue=sum((ForwardRates/Period).*DiscountFactors);
SwapFixedRates=Period*PresentValue/sum(DiscountFactors);
ForwardRates,DiscountFactors,PresentValue,SwapFixedRates,

x=BondData(:,1);
x=datenum(x);
y=ForwardRates;
plot(x,y)
title('Implied Forward Curve');
xlabel('Maturite Dates for Bonds');
ylabel('Coupon Rates');
```

```
>>

ZeroRates =

    0.0715
    0.0743
    0.0760
    0.0785
    0.0803
    0.0827
    0.0856
    0.0862
    0.0866
    0.0944
```

ZeroRates = zbtprice([Maturity CouponRate], Prices, Settle)

Zero Rates: a bond in which no periodic coupon is paid over the life of the contract. Both the principal and the interest rate are paid at the maturity date.

zbtprice: zero curve bootstrapping from coupon bond date given price.

```
DiscountFactors =

    0.9658
    0.9297
    0.8942
    0.8571
    0.8215
    0.7839
    0.7459
    0.7133
    0.6829
    0.6302
```

ForwardRates = zero2fwd(ZeroRates,Maturity,Settle)

Forward Rates: a projection on future interest rates calculated from either the spot rates or the yield curve.

zero2fwd: forward curve given a zero curve.

Zero Rates: a number of bonds-by-1 vectors of annualised zero rates, as decimal fractions. In aggregate, the rates constitute an implied zero curve for the investment horizon represented by curve dates.

```
DiscountFactors =

    0.9658
    0.9297
    0.8942
    0.8571
    0.8215
    0.7839
    0.7459
    0.7133
    0.6829
    0.6302
```

DiscountFactors = zero2disc(ZeroRates,Maturity,Settle)

zero2disc : discount curve given a zero curve.

Zero Rates: a number of bonds-by-1 vectors of annualised zero rates, as decimal fractions. In aggregate, the rates constitute an implied zero curve for the investment horizon represented by curve dates.

```
PresentValue =

    0.3694
```

The swap's price is calculating by equating the present value of the fixed cash flows with the present value of the cash flows derived from the implied forward rates.

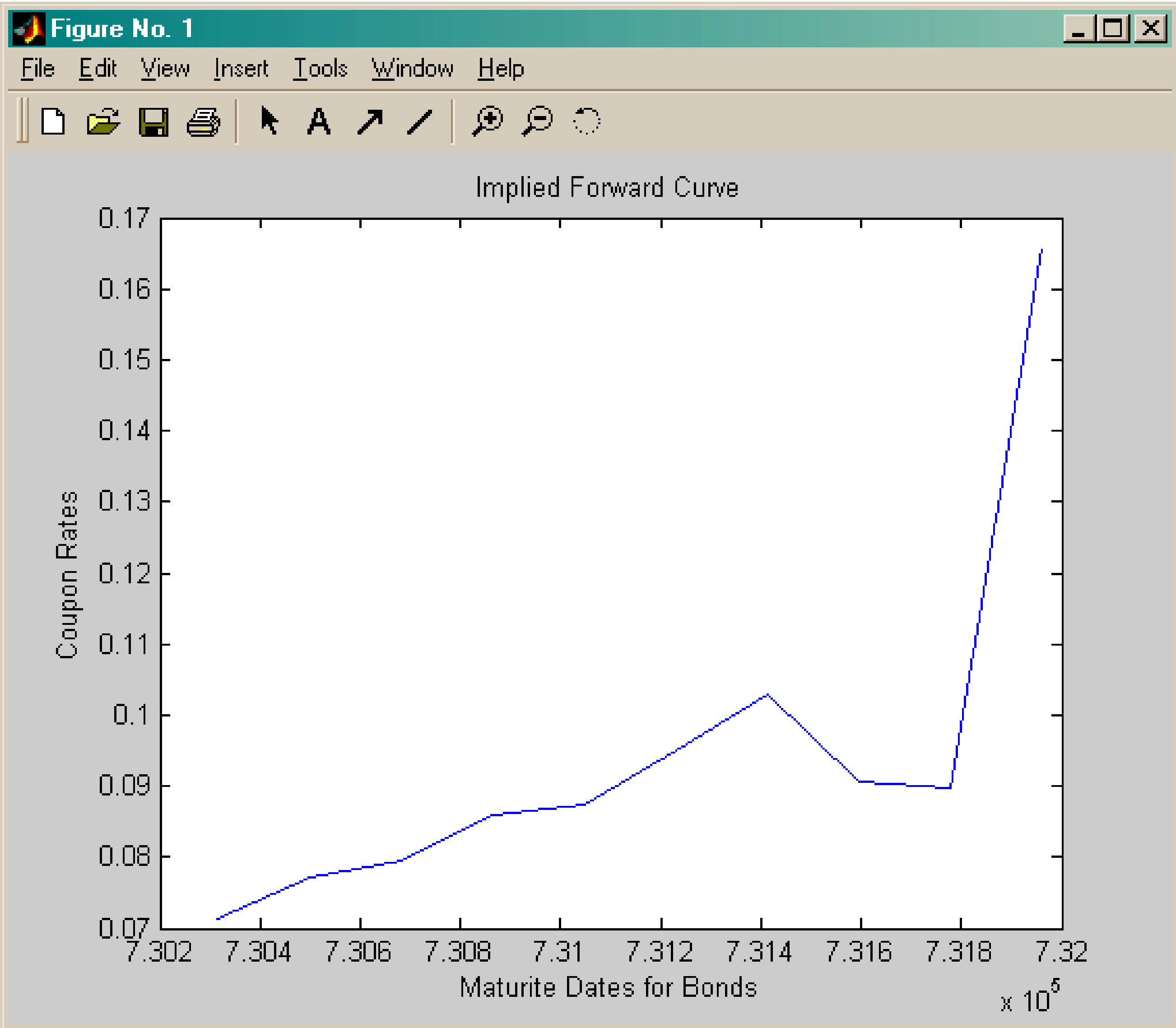
```
SwapFixedRates =

    0.0921
>>
```

The swap price is 9.21%, would likely be the mid-point between a market maker's bid/ask quotes.

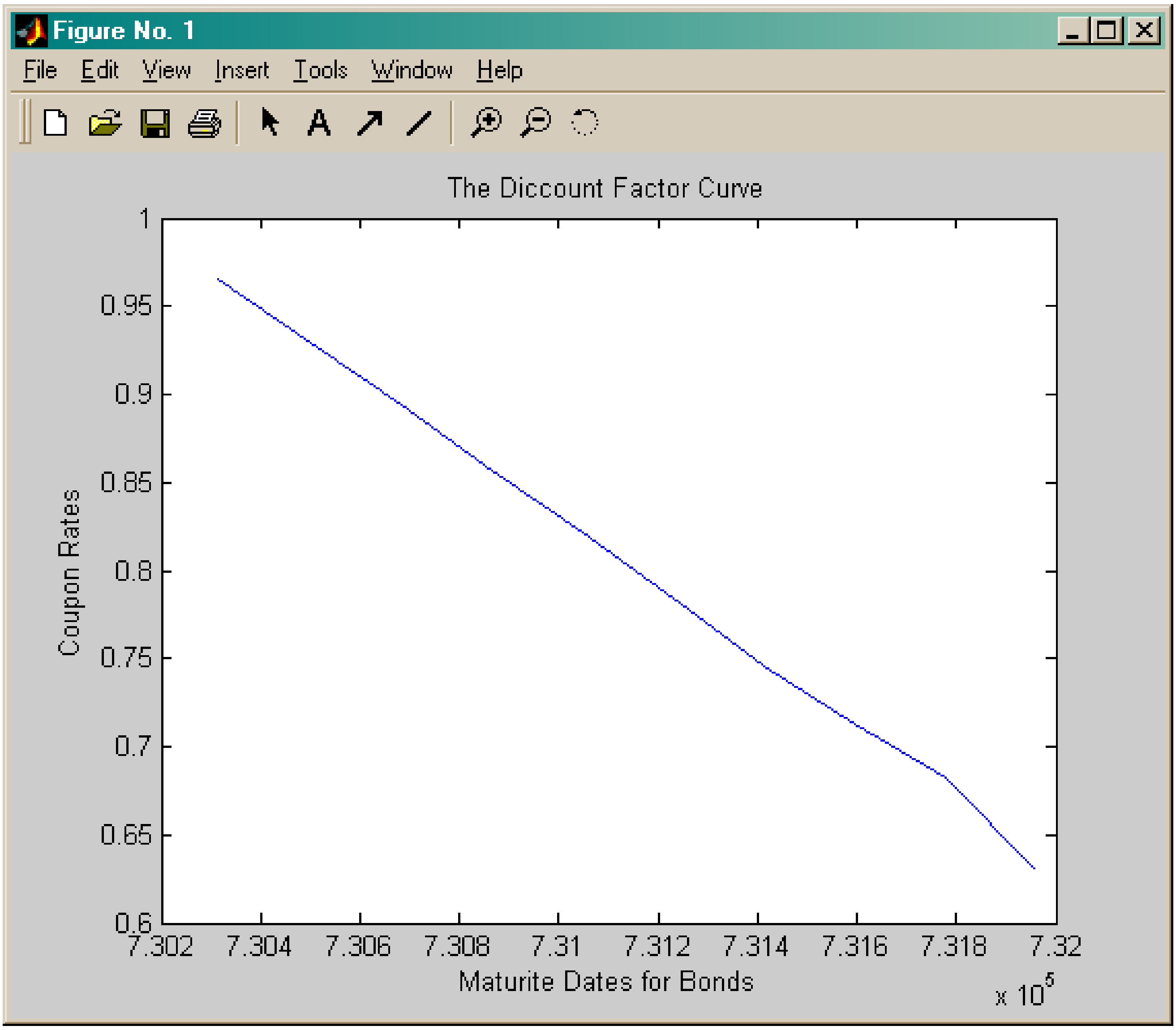
Graph number 1. Implied Forwards Curve

Forward Rates derived from the Swap curve.



Graph number 2. The Discount Curve

The Discount Curve is the relationship between the coupon rate and the Maturity dates.



Conclusions

Risk managers and speculators can use derivatives for their specific ends, and many traders have been attracted to financial derivatives because of their trading efficiency, particularly they low transaction costs and highly liquid markets.

Swaps are one of the major innovations of the 80s. Remember a swap it's a contract that involves the exchange of cash flows according to a formula that depends on the value of one or more markets variables.

The principal in a swap agreement can be varied throughout the term of the swap to meet the needs of a counterparty, amortizing swap, step-up swap, deferred swaps, forward swaps.

Financial derivatives have attained their overwhelming popularity and rapid growth because its, brings benefits to the market and help to make markets more nearly complete.

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